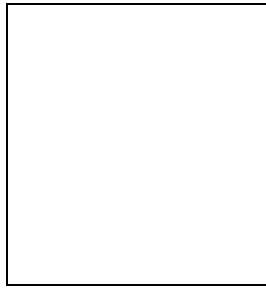


**SCALAR-TENSOR THEORIES AND COSMOLOGY
&
TESTS OF A QUINTESSENCE-GAUSS-BONNET COUPLING^a**

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Scalar-tensor theories are the best motivated alternatives to general relativity and provide a mathematically consistent framework to test the various observable predictions. They can involve three functions of the scalar field: (i) a potential (as in “quintessence” models), (ii) a matter–scalar coupling function (as in “extended quintessence”, where it may also be rewritten as a nonminimal coupling of the scalar field to the scalar curvature), and (iii) a coupling function of the scalar field to the Gauss-Bonnet topological invariant. We recall the main experimental constraints on this class of theories, and underline that solar-system, binary-pulsar, and cosmological observations give qualitatively different tests. We finally show that the combination of these data is necessary to constrain the existence of a scalar–Gauss-Bonnet coupling.

1 Introduction

In the most natural alternative theories to general relativity (GR), gravity is mediated not only by a (spin-2) graviton corresponding to a metric $g_{\mu\nu}$, but also by a (spin-0) scalar field φ . Such scalar partners generically arise in all extra-dimensional theories, and notably in string theory. A dilaton is indeed already present in the supermultiplet of the 10-dimensional graviton, and several other scalar fields (called the moduli) also appear when performing a Kaluza-Klein dimensional reduction to our usual spacetime. They correspond to the components of the metric tensor g_{mn} in which m and n label extra dimensions. Moreover, contrary to other alternative theories of gravity, scalar-tensor theories respect most of GR’s symmetries: conservation laws, constancy of non-gravitational constants, and local Lorentz invariance even if a subsystem is influenced by external masses. They can also satisfy exactly the weak equivalence principle (universality of free fall of laboratory-size objects) even for a massless scalar field.

^aContribution to the XXXVIIth Rencontres de Moriond on *Gravitational Waves and Experimental Gravity*, Les Arcs (France), March 23–29 2003.

Scalar fields are also involved in the cosmological models which reproduce most consistently present observational data. In particular, inflation theory is based on the presence of a scalar φ in a potential $V(\varphi)$ (for instance parabolic). It behaves as a fluid with a positive energy density $8\pi G\rho_\varphi = \dot{\varphi}^2 + 2V(\varphi)$ but a negative pressure $8\pi Gp_\varphi = \dot{\varphi}^2 - 2V(\varphi)$. This causes a period of exponential expansion of the universe, which can explain why causally disconnected regions at present may have been connected long ago. The isotropy of the observed cosmic microwave background (CMB) can thus be understood. Inflation also predicts that our universe is almost spatially flat, just because any initial curvature has been exponentially reduced by the expansion. This is in remarkable agreement with the location of the first acoustic peak of the CMB spectrum at a multipolar index¹ $\ell \simeq 220$. Observations of type Ia supernovae^{2,3} tell us that there is about 70% of negative-pressure dark energy in our present universe ($\Omega_\Lambda \simeq 0.7$), suggesting that its expansion has been re-accelerating recently (since redshifts $z \sim 1$). This can be explained by the presence of a cosmological constant Λ in GR, but the quantity $\Omega_\Lambda \simeq 0.7$ translated in natural units gives an extremely small value $\Lambda \simeq 3 \times 10^{-122} c^3 / (\hbar G)$, very problematic for particle physics if Λ is to be interpreted as the vacuum energy. This is the main reason why “quintessence” models have been proposed, in which the cosmological constant is replaced again by the potential $V(\varphi)$ of a scalar field. Its evolution towards a minimum of V during the cosmological expansion then explains more naturally why the present value $V(\varphi_0) \simeq \Lambda/2$ is so small.

Besides these theoretical and experimental reasons for studying scalar-tensor theories of gravity, one of their greatest interests is to embed GR within a class of mathematically consistent alternatives, in order to understand better which theoretical features have been experimentally tested, and which can be tested further.

The following action defines the most general theory satisfying the weak equivalence principle and involving only one spin-0 degree of freedom besides the usual (spin-2) graviton:

$$\begin{aligned} S = & S_{\text{matter}}[\text{matter}; \tilde{g}_{\mu\nu} \equiv A^2(\varphi)g_{\mu\nu}] \\ & + \frac{c^3}{4\pi G} \int \sqrt{-g} \left\{ \frac{R}{4} - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) \right\} \\ & - \hbar \int \sqrt{-g} W(\varphi) (R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2) , \end{aligned} \quad (1)$$

where $\tilde{g}_{\mu\nu}$ is the metric to which matter is universally coupled, and $g_{\mu\nu}$ is the Einstein metric (describing the spin-2 degree of freedom). This action involves three function of the scalar field: a coupling function $A(\varphi)$ to matter, a potential $V(\varphi)$, and a coupling function $W(\varphi)$ to the Gauss-Bonnet topological invariant. Any other combination of the curvature tensor would introduce an extra scalar field in the theory, and/or a second negative-energy massive graviton which would make the model unstable.

In Sections 2 and 3, we will not consider any scalar–Gauss–Bonnet coupling, and set $W(\varphi) = 0$. The potential $V(\varphi)$ will also be neglected in Section 2, in which we will review the main experimental constraints on the coupling function $A(\varphi)$, coming from solar-system and binary-pulsar data.^{4,5} In Section 3, we will summarize our results concerning the reconstruction of $A(\varphi)$ and $V(\varphi)$ from cosmological observations.^{7,8} Section 4 will be devoted to the scalar–Gauss–Bonnet coupling $W(\varphi)$, which can be constrained only if one takes into account both solar-system and cosmological data.¹⁰ We will finally give our conclusions in Section 5.

2 Solar-system and binary-pulsar constraints

The effects of a massive scalar field has a negligible effect on the motion of celestial bodies if its mass is large with respect to the inverse of the interbody distances. On the other hand, if its

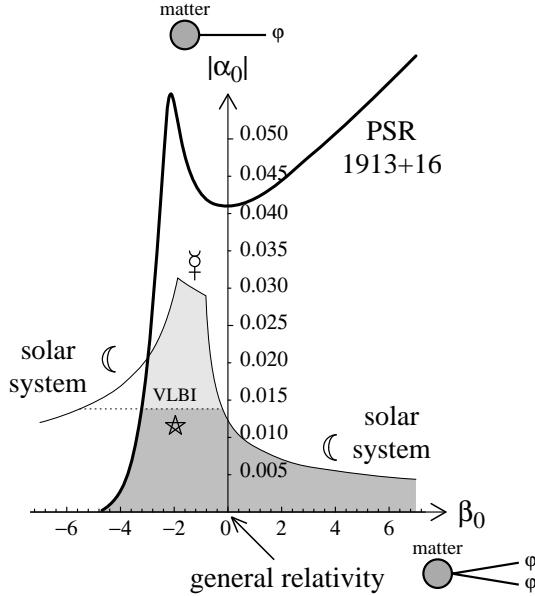


Figure 1: Solar-system and binary-pulsar constraints on the matter-scalar coupling function $\ln A(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2 + O(\varphi - \varphi_0)^3$. The allowed region is shaded. The vertical axis ($\beta_0 = 0$) corresponds to Brans-Dicke theory with a parameter $2\omega_{\text{BD}} + 3 = 1/\alpha_0^2$. The horizontal axis ($\alpha_0 = 0$) corresponds to theories which are perturbatively equivalent to GR, i.e., which predict strictly no deviation from it (at any order $1/c^n$) in the weak-field conditions of the solar system.

mass is small enough, its potential $V(\varphi)$ can be locally neglected, but its coupling function to matter, $A(\varphi)$, is strongly constrained by experiment.

The predictions of metric theories of gravity in weak-field conditions can be parametrized by a set of 10 real numbers in the so called “PPN” formalism (parametrized post-Newtonian). All of them are presently constrained to be very close to their general relativistic values, and in particular the two famous Eddington parameters β and γ (both equal to 1 in GR). In scalar-tensor theories,^{4,5} they are related to the first two derivatives of $\ln A(\varphi)$, computed at the background value φ_0 of the scalar field. They give the constraints displayed as a thin line in Figure 1 (where the Moon symbol refers to Lunar Laser Ranging, the Mercury symbol to the perihelion shift of this planet, and the star symbol to light deflection as measured by Very Long Baseline Interferometry). Solar-system tests thus constrain the first derivative $\alpha_0 \equiv \partial \ln A(\varphi)/\partial \varphi$ to be small, but do not tell us much about the second derivative $\beta_0 \equiv \partial^2 \ln A(\varphi)/\partial \varphi^2$. If α_0 is small enough, arbitrary large positive or negative values of β_0 are a priori allowed.

Binary-pulsar give qualitatively different constraints because of nonperturbative strong-field effects. Indeed, the largest deviations from the flat metric, at the surface of a star, are of order $GM/Rc^2 \simeq 0.2$ for a pulsar (neutron star), as compared to 2×10^{-6} for the Sun. We showed^{4,5} that if β_0 is negative, then it is energetically favorable for a neutron star, above a critical mass, to create a nonvanishing scalar field. Since this is analogous to the spontaneous magnetization of ferromagnets, we called this effect “spontaneous scalarization”. Such macroscopic scalar charges change drastically the physics of a binary system, notably because it emits dipolar gravitational (scalar) waves $\propto 1/c^3$, much larger than the usual quadrupolar radiation $\propto 1/c^5$ predicted by GR. This is the reason why binary-pulsar data, which are consistent with GR, rule out scalar-tensor models such that $\beta_0 < -5$, even for vanishingly small values of α_0 (i.e., even if they are strictly indistinguishable from GR in the solar system).

We also showed that the LIGO/VIRGO interferometers will be more sensitive to β_0 than solar-system tests, but binary-pulsar data are so precise that they already exclude the models which predict significant effects in the gravitational waveforms. This is a good news, since it

proves that pure GR wave templates suffice to analyze future LIGO/VIRGO data. On the other hand, it was shown⁶ that the LISA interferometer can be sensitive to scalar effects which are still allowed by all present tests.

In conclusion, solar-system tests tightly constrain the first derivative of $\ln A(\varphi)$ (linear matter-scalar coupling strength), whereas binary-pulsar data impose that its second derivative (quadratic coupling matter-scalar-scalar) is not large and negative. We will now see that cosmological observations give access to the full shape of this coupling function, of course not with the same accuracy as the above tests, but with the capability of constraining any higher derivative of $\ln A(\varphi)$ (vertex of matter with any number of scalar lines). Moreover, cosmological data can also give access to the full shape of the potential $V(\varphi)$.

3 Reconstruction of a scalar-tensor theory from cosmological observations

In cosmology, the usual approach to study quintessence models is to assume a particular form for the potential $V(\varphi)$ (and the matter-scalar coupling function $A(\varphi)$ when one considers “extended quintessence” models), to compute all possible observable predictions, and to compare them to experimental data.

In contrast, in the phenomenological approach, one wishes to *reconstruct* the Lagrangian of the theory from cosmological observations. We proved⁷ that the knowledge of the luminosity distance $D_L(z)$ and of the density fluctuations $\delta_m(z) = \delta\rho/\rho$ as functions of the redshift z indeed suffices to reconstruct both the potential $V(\varphi)$ and the coupling function $A(\varphi)$. Although the explicit reconstruction needs some algebra, this result seems anyway obvious: It is possible to *fit* two observed functions [$D_L(z)$ and $\delta_m(z)$] thanks to two unknown ones [$V(\varphi)$ and $A(\varphi)$].

However, future experiments (like the SNAP satellite) will only give access to the luminosity distance $D_L(z)$ with a good accuracy, and the density contrast $\delta_m(z)$ cannot yet be used to constrain the models. A *semi-phenomenological* approach can thus be useful: We make some theoretical hypotheses on either the potential $V(\varphi)$ or the coupling function $A(\varphi)$, and we reconstruct the other one from $D_L(z)$. A priori, one may think that such a reconstruction is again obvious: We fit one observed function [$D_L(z)$] with one unknown function [$V(\varphi)$ or $A(\varphi)$]. But this naive reasoning is only valid locally, on a small interval. Indeed, the reconstructed function may for instance diverge for some value of the redshift, or one of the degrees of freedom may need to take a negative energy beyond a given redshift, which would make the theory unstable (and ill defined as a field theory on the surface where the energy changes its sign). The positivity of the graviton energy implies $A^2(\varphi) > 0$, which can be translated in terms of the standard Brans-Dicke scalar field as $\Phi_{\text{BD}} > 0$. On the other hand, the positivity of the scalar-field energy imposes the minus sign in front of the scalar kinetic term $-(\partial_\mu\varphi)^2$ in action (1), which translates as $\omega_{\text{BD}} > -\frac{3}{2}$ in terms of the standard Brans-Dicke parameter. We showed that these conditions impose tight constraints on the theories as soon as one knows $D_L(z)$ over a wide enough interval $z \in [0, \sim 2]$.

For instance, we proved that the present accelerated expansion of the universe can be perfectly described by a scalar-tensor theory with a vanishing potential $V(\varphi) = 0$ (and therefore a vanishing cosmological constant too). We derived analytically the coupling function $A(\varphi)$ which reproduces exactly the same evolution of the scale factor $a(z)$ as the one predicted by GR plus a cosmological constant. We even found that the reconstructed function $\ln A(\varphi)$ has a nice parabolic shape, with a minimum very close to the present value φ_0 of the scalar field, and a positive second derivative. This is not only consistent with binary-pulsar data (which forbid large and negative values of this second derivative) but also with the cosmological attractor phenomenon analyzed by Damour and Nordtvedt⁹: The scalar field is generically attracted towards a minimum of $\ln A(\varphi)$ during the cosmological expansion, whereas some fine tuning would be necessary to reach a maximum (negative second derivative). Therefore, we are in the difficult

situation in which two very different theories are both consistent with experimental data, and there seems to be no way to distinguish them. Fortunately, we found that this scalar-tensor theory cannot mimic GR plus a cosmological constant beyond a redshift $z \sim 0.7$, because the scalar field φ would diverge at this value, and above all because the graviton energy would become negative beyond. Therefore, it suffices to measure $D_L(z)$ precisely enough up to $z \sim 1$ to rule out such a potential-free scalar-tensor theory. Actually, if $D_L(z)$ is measured over a wider interval $z \in [0, \sim 2]$, we showed that large experimental errors (tens of percent) were not problematic: It is still possible to distinguish this potential-free model from GR plus a cosmological constant, and thereby to rule out one of them. The results of the SNAP satellite up to $z \sim 2$ will therefore be very useful to constrain scalar-tensor theories of gravity.

One can also impose a particular form of the coupling function $A(\varphi)$ and reconstruct the potential $V(\varphi)$ which reproduces the observed luminosity distance $D_L(z)$. For instance, for a minimally coupled scalar field $A(\varphi) = 1$ (usual “quintessence”) in a spatially curved universe, we analytically derived the expression of $V(\varphi)$ which gives the same cosmological evolution as GR plus a cosmological constant in a spatially flat universe. We found that the shape of the potential is smoother when the universe is (marginally) closed. If it is flat or almost flat, one obviously recovers a cosmological constant with its unnaturally small value $\Lambda \simeq 3 \times 10^{-122} c^3 / (\hbar G)$. Therefore, in that case, aesthetic reasons may help us discriminate between the theories, instead of the much stronger argument of the positivity of energy that we used above. This shows anyway that the sole knowledge of $D_L(z)$ suffices to constrain scalar-tensor theories of gravity.

4 Experimental constraints on a scalar–Gauss–Bonnet coupling

In order to illustrate the different kinds of experimental constraints that can be imposed¹⁰ on the scalar–Gauss–Bonnet coupling function $W(\varphi)$, we will now focus of a theory with $A(\varphi) = 1$ and $V(\varphi) = 0$ in action (1).

Solar-system and binary-pulsar tests are local, and any deviation from GR depends on the magnitude of the scalar field created by a massive body. Let us thus analyze first the equation satisfied by φ in the vicinity of a spherical mass M_\odot . We can assume that the metric is close to the Schwarzschild solution, and we get at the first nonvanishing order in powers of GM_\odot/c^2

$$\square\varphi = \frac{3r_0^2}{r^6} \left(\frac{2GM_\odot}{c^2} \right)^2 [W'_0 + W''_0\varphi + O(\varphi^2)] , \quad (2)$$

where we have set $r_0^2 \equiv 16\pi G\hbar/c^3$, and where the derivative $W'(\varphi)$ has been expanded in powers of φ in the right-hand side. Since we are assuming that φ takes small values, let us neglect the contribution $W''_0\varphi$. We can then compute any observable prediction, but we quote below only the results for the light deflection angle ($\Delta\theta_*$) and for the perihelion shift per orbit ($\Delta\theta_p$), which suffice for our purpose:

$$\Delta\theta_* = \frac{4GM_\odot}{\rho_0 c^2} + \frac{1536}{35} \left(\frac{GM_\odot}{\rho_0 c^2} \right)^3 \left(\frac{r_0}{\rho_0} \right)^4 W'^2_0 , \quad (3)$$

$$\Delta\theta_p = \frac{6\pi GM_\odot}{pc^2} + 192\pi \left(\frac{GM_\odot}{pc^2} \right)^2 \left(\frac{r_0}{p} \right)^4 W'^2_0 , \quad (4)$$

where ρ_0 is the minimal distance between the light ray and the Sun, and p is the *semilatus rectum* of an orbit. The first terms on the right-hand sides are the usual general relativistic predictions, at first order in GM_\odot/c^2 . In conclusion, solar-system (and binary-pulsar) tests can easily be passed if $|W'_0|$ is small enough.

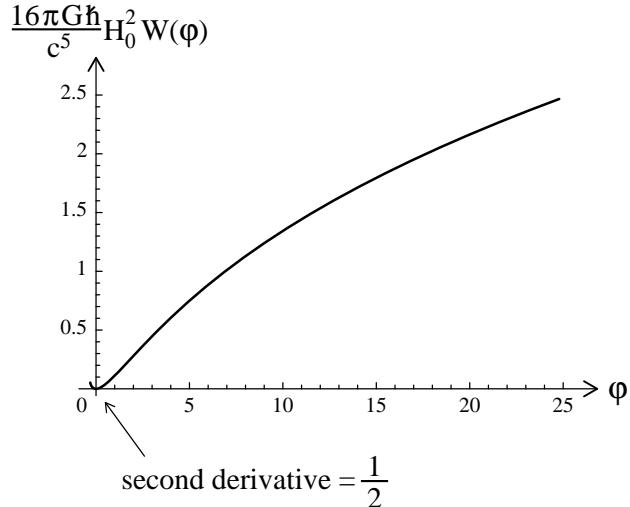


Figure 2: Scalar–Gauss–Bonnet coupling function $W(\varphi)$ which exactly reproduces the cosmological expansion predicted by GR plus a cosmological constant.

One can now reconstruct the full shape of $W(\varphi)$ from the cosmological observation of the luminosity distance $D_L(z)$, as in the previous section. We found that this can always been done, without any problem of negative energy, contrary to what we saw in Section 3. Moreover, there exists again an attraction mechanism which drives the scalar field towards a minimum of $W(\varphi)$ during the cosmological expansion. Therefore, a small value of the slope $|W'_0|$ is indeed expected at present, consistently with what is needed for solar-system tests. In conclusion, we are faced again with a serious problem: We just found a theory which seems to be consistent with all experimental data, although it is very different from GR in its field content. Our aim is therefore to find a way to distinguish it from GR, or to rule it out for internal consistency reasons.

Figure 2 displays this reconstructed coupling function $W(\varphi)$, in which the present value of the scalar field is close to the minimum at $\varphi = 0$. Its shape is nicely smooth, but its second derivative at the origin is huge if one divides it by the tiny factor $(16\pi G \hbar / c^5) H_0^2$, where H_0 denotes the Hubble constant. One gets $W''_0 \simeq 7 \times 10^{119}$, which is in fact not surprising, since the coupling function $W(\varphi)$ behaves in action (1) as the inverse of a cosmological constant. Indeed, $W(\varphi)$ multiplies the square of the curvature tensor, whereas the usual Einstein-Hilbert term involves the first power of the scalar curvature, and a cosmological constant does not multiply any curvature term at all. Therefore, it was expected that $W(\varphi)$ involve a dimensionless number of the order of the inverse of $(\hbar G / c^3) \Lambda \simeq 3 \times 10^{-122}$. Therefore, this model is ugly, but it is not yet ruled out. One should not confuse fine tuning and large (or small) dimensionless numbers in a model. We are here in the second situation, but there is a priori no fine tuning since the scalar field is attracted towards the minimum of $W(\varphi)$ during the cosmological expansion. There remains to study how efficiently it is attracted, but this is actually not necessary for our purpose.

Indeed, W''_0 takes such a gigantic value that an approximation that we made to analyze solar-system tests is no longer valid. Indeed, we have $|W''_0 \varphi| \gg |W'_0|$, so that the second term on the right-hand side of Eq. (2) cannot be neglected. To simplify the discussion, we will anyway assume that $W(\varphi)$ is parabolic, which is a good approximation in a vicinity of the minimum $\varphi = 0$. We will thus neglect the higher order terms $O(\varphi^2)$ in Eq. (2); taking them into account would not change our conclusions below. We did not find a close analytic solution to Eq. (2),

but it is possible to write it as a series

$$\varphi = \frac{W'_0}{W''_0} \sum_{n \geq 1} \frac{1}{(3 \times 4)(7 \times 8) \cdots (4n-1)(4n)} \left(\frac{12r_0^2 G^2 M_\odot^2 W''_0}{r^4 c^4} \right)^n \quad (5)$$

$$\simeq \frac{W'_0}{W''_0} \begin{cases} \cos \left(\frac{GM_\odot r_0}{r^2 c^2} \sqrt{3|W''_0|} \right) - 1 & \text{if } W''_0 < 0, \\ \cosh \left(\frac{GM_\odot r_0}{r^2 c^2} \sqrt{3|W''_0|} \right) - 1 & \text{if } W''_0 > 0. \end{cases} \quad (6)$$

The second expression is a good approximation if the argument of the cosine (or hyperbolic cosine if $W''_0 > 0$) is much greater than 1. This is the case if we use the huge value of W''_0 obtained above from the cosmological reconstruction, and a typical solar-system distance for the radius r : The argument of the hyperbolic cosine is then of order 10^8 .

The above solution is such that $\varphi \propto W'_0$, therefore we do not find any nonperturbative effect similar to the “spontaneous scalarization” of neutron stars mentioned in Section 2 above. Moreover, $\varphi \rightarrow 0$ as $r \rightarrow \infty$, and we recover GR for distances $r > 4 \times 10^{14}$ m (i.e., farther than the solar system including Oort’s comet cloud). On the other hand, there are highly nonlinear corrections proportional to $1/r^{4n}$ within the solar system. Since the ratio $(12r_0^2 G^2 M_\odot^2 W''_0 / r^4 c^4)$ is much greater than 1, its successive powers blow up, but they are compensated by the factors $1/(3 \times 4 \times 7 \times \cdots 4n)$ which behave like the inverse of factorials. Therefore, the successive terms of series (5) start to grow exponentially, then reach a maximum for a value of the index n which may be large, and finally tend towards zero. Each of these successive terms must be assumed to be small enough for the model to pass all classical tests, but one should not forget that the largest one does not correspond to $n = 1$.

In order to study the effects of such highly nonlinear terms in the solar system, we compute their corrections to the Schwarzschild metric in the form

$$ds^2 = - \left(1 + \sum_{n \geq 1} \frac{\beta_n}{\rho^n} \right) c^2 dt^2 + \left(1 + \sum_{n \geq 1} \frac{\alpha_n}{\rho^n} \right) d\rho^2 + \rho^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (7)$$

and we find that the light deflection angle and the perihelion shift are respectively given by

$$\Delta\theta_* = \sum_{n \geq 1} 2^{n-1} \frac{\Gamma(\frac{n+1}{2})^2}{\Gamma(n+1)} \frac{\alpha_n - n\beta_n}{\rho_0^n} + O(\alpha_n, \beta_n)^2, \quad (8)$$

$$\Delta\theta_p = \frac{6\pi GM_\odot}{pc^2} - \sum_{n \geq 3} \frac{n(n-1)\beta_n c^2}{2GM_\odot p^{n-1}} \pi + O(\alpha_n, \beta_n)^2, \quad (9)$$

which generalize Eqs. (3)-(4) above. Note that these results are (independently) perturbative in each of the coefficients α_n and β_n , because we are assuming that the scalar-field effects are negligible with respect to the general relativistic predictions. However, the dominant scalar-field corrections may correspond to a large value of index n .

When solution (5) or its approximation (6) are used to compute the metric coefficients α_n and β_n in the above observable predictions, and if we use the huge value of W''_0 obtained from the previous cosmological reconstruction of $W(\varphi)$, we get the following experimental constraint:

$$|W'_0| < 10^{-2 \times 10^{11}}. \quad (10)$$

Now we can speak of fine tuning, and even of *hyperfine* tuning! This constraint simply means that the present value of the scalar field must be *exactly* at the minimum of the coupling function $W(\varphi)$, otherwise solar-system tests are violated. And since the universe is still evolving, the scalar field cannot remain so close to the minimum for more than a fraction of a second. Therefore, even if we assumed that $W'_0 = 0$ strictly to pass solar-system tests, this would not

be the case a tiny instant later. In conclusion, we managed to rule out the scalar-tensor model $A(\varphi) = 1$, $V(\varphi) = 0$ and $W(\varphi) \neq 0$. It cannot describe an accelerating expansion of the universe at present and pass solar-system (and binary-pulsar) tests at the same time.

Of course, this result does not rule out any scalar–Gauss–Bonnet coupling. A model with three (or even two) free functions $A(\varphi)$, $V(\varphi)$ and $W(\varphi)$ can obviously pass all present tests. For instance, GR plus a cosmological constant simply corresponds to $A(\varphi) = 1$, $V(\varphi) = \Lambda/2$ and $W(\varphi) = 0$. But the presence of a non-constant coupling $W(\varphi)$ can change the physics at small scales, notably in the very early universe (Big-Bang) and for later clustering properties.

The fact that $W(\varphi)$ induces effects at small scales can be understood by a simple dimensional argument. Since this function multiplies the square of the curvature in action (1), it induces corrections proportional to $1/r^7$ (and higher orders) to the Newtonian potential in $1/r$, and thereby generically dominates at small scales. However, we saw above that this quick reasoning can be erroneous in some perturbative but highly nonlinear situations. Indeed, if W_0'' takes very large and *negative* values, the cosine involved in Eq. (6) shows that φ is always of the order of $-W_0'/W_0''$, even for small distances r . One can then prove that the (very easily satisfied) condition $|r_0^2 W_0'| \ll r^2$ suffices for all scalar-field effects to be negligible in the solar system, even if $|W_0''| \sim 10^{120}$. This remark underlines that nonlinear effects can drastically change the intuitive behavior, but let us recall that our cosmological reconstruction above predicted a large and *positive* value for W_0'' . In that case, we did find that the scalar–Gauss–Bonnet coupling induces large effects at small scales, and even exponentially larger than the linear results (3)–(4).

5 Conclusions

Scalar-tensor theories of gravity are the best motivated alternatives to general relativity. Three classes of experimental data give *qualitatively* different constraints on them. Solar-system tests strongly constrain the first derivative of the matter-scalar coupling function $A(\varphi)$ (i.e., the linear matter-scalar coupling strength). Binary-pulsar data forbid large and negative values of its second derivative (quadratic matter-scalar-scalar coupling). The knowledge of the two cosmological functions $D_L(z)$ and $\delta_m(z)$ suffices to reconstruct the full shape of both $A(\varphi)$ and the potential $V(\varphi)$ on a finite interval of φ . The knowledge of the luminosity distance $D_L(z)$ alone over a *wide* redshift interval strongly constrains the theories if one takes into account solar-system (and binary-pulsar) data, the positivity of the graviton and scalar energies, and the stability and naturalness of the models. Future data, provided by experiments like the SNAP satellite, will notably allow us to discriminate between GR plus a cosmological constant and a potential-free scalar-tensor theory. The possible coupling $W(\varphi)$ of the scalar field to the Gauss–Bonnet topological invariant can be constrained only if one takes into account cosmological and solar-system data together. The predictions of the model at small distances can depend on highly nonlinear corrections. Of course, a model including all three functions $A(\varphi)$, $V(\varphi)$ and $W(\varphi)$ is experimentally allowed, since GR plus a cosmological constant is a particular case. The presence of a scalar–Gauss–Bonnet coupling $W(\varphi)$ will generically change the behavior of the theory at small scales (clustering, Big Bang).

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